Review: Informal Sequence Convergence - 10/3/16

1 Monotone Sequences

Definition 1.0.1 *A sequence $\{a_n\}$ is **monotone** if either $a_n \leq a_{n+1}$ for all n, or $a_n \geq a_{n+1}$ for all n.*

Example 1.0.2 $\{a_n\} = \{1, 2, 3, 4, 4, 4, 5, 6, 7, \dots\}$ *is monotonically increasing.*
$\{b_n\} = \{5, 4, 3, 3, 2, 1, 0, -1, \dots\}$ *is monotonically decreasing.*
$\{c_n\} = \{1, 2, 3, 4, 5, \dots\}$ *is monotonically increasing.*
$\{d_n\} = \{\frac{1}{n}\}$ *is monotonically decreasing.*
$\{e_n\} = \{\frac{(-1)^n}{n}\}$ *is NOT monotonic.*
$\{f_n\} = \{1, 1, 1, 1, \dots\}$ *is BOTH monotonically increasing AND monotonically decreasing.*

2 Limits of Sequences

A limit of a sequence is a number L that the terms of the sequence get close to as we write down more terms. The notation is $\lim_{n\to\infty} a_n = L$. In this case, we say that the sequence converges to L.

Example 2.0.3 *Does $\{\frac{1}{n}\}$ converge? As we plug in larger values for n, the fraction gets smaller and smaller. The best we can do right now is guess that the sequence converges to zero.*

Example 2.0.4 *Does $\{\frac{n+1}{n^2}\}$ converge? If so, what is it's limit? The n^2 is getting bigger a lot faster than the $n+1$. This means that overall, the sequence is decreasing. We can guess that the limit is zero.*

Example 2.0.5 $\{(-1)^n\}$ *does not converge.*
$\{\frac{(-1)^n)}{n}\}$ *converges to zero.*

Example 2.0.6 $\{\sin(n)\}$ *oscillates in between -1 and 1. It does not converge.*
$\{\cos(1/n)\}$ *converges to 1. Since $\frac{1}{n}$ gets closer and closer to zero, then $\cos(\frac{1}{n})$ gets closer and closer to* $\cos(0) = 1$.

Practice Problems Do the following sequences converge? If so, what to?

1. $\{\frac{n^2}{3}\}$
2. $\{n - n^2\}$
3. $\{\frac{2n+1}{2n-1}\}$
4. $\{\ln(\cos(\frac{1}{n}))\}$

Solutions

1. The sequence does not converge, since n^2 just keeps getting bigger.
2. The sequence does not converge: since n^2 gets bigger so much faster than n, the entire sequence goes to $-\infty$.
3. The sequence converges to 2. Try plugging in really large numbers.
4. The sequence converges to 0. Remember from the example, $\{\cos(\frac{1}{n})\}$ gets closer and closer to 1. Thus $\ln(\cos(\frac{1}{n}))$ gets closer and closer to $\ln(1)$, which is 0.